ECE 343 Signals and Systems

Homework 4: Convolution (on $\mathbb{R}$)

1. Let $x(t) = \cos(\omega t)u(t)$ and $h(t) = \sin(\omega t)u(t)$. Find $y(t) = (x * h)(t)$.

2. Show that

$$\int_{-\infty}^{\infty} (x * y)(t)dt = \left(\int_{-\infty}^{\infty} x(t)dt\right)\left(\int_{-\infty}^{\infty} y(t)dt\right)$$

   Assume that all of the integrals converge.

3. Let $x(t) = \Pi(t+2) + \Pi(t) + \Pi(t-2)$.

   (a) Sketch $x(t)$.

   (b) Find $y(t) = (x * x)(t)$.

   (c) Sketch $y(t)$.

4. In class, there was an example of convolution failing to be associative, but one of the "functions" in the example was the strange $\delta'(t)$. Here is an example with nicer functions.

   (a) Show that convolution is associative, assuming that Fubini's Theorem applies. Fubini's Theorem says that if

$$\int_{\mathbb{R}^2} |h(u,v)|dA < \infty$$

   then

$$\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} h(u,v)du\right)dv = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} h(u,v)dv\right)du = \int_{\mathbb{R}^2} h(u,v)dA$$

   In other words, assume that iterated integrals can be evaluated in either order.

   (b) Let

$$\begin{aligned} x_1(t) &= u(t) \\ x_2(t) &= \frac{d}{dt}(e^{-t^2}) = -2te^{-t^2} \\ x_3(t) &= u(-t) \end{aligned}$$

   Show $x_1 * (x_2 * x_3) \neq (x_1 * x_2) * x_3$.